Problem - 3 :- The origins of two systems S and S' coincide initially. The system S' is moving with velocity $(3\hat{i} + 4\hat{j} + 6\hat{k})$ cm/sec. relative to S. After 2 seconds if the coordinates of any point as observed by an observer at the origin of S are $(5, 6, -9)$ cm. Find the co-ordinates of that point relative to an observer at the origin of S'.

Sol :- From Galilean transformation equ^{ns}, we have

$$x' = x - v_x t,$$
$$y' = y - v_y t,$$
$$z' = z - v_z t$$

Here $x = 5,\ y = 6,\ z = -9,\ t = 2$ seconds
and $v_x = 3,\ v_y = 4,\ v_z = 4$ cm/sec.

$\therefore$ $x' = 5 - 3 \times 2 = -1$
$y' = 6 - 4 \times 2 = -2$
$z' = -9 - 6 \times 2 = -9 - 12 = -21$

The co-ordinates of the point relative to an observer at the origin of S' are $(-1, -2, -21)$ cm.

Problem - 4 :- Use Galilean transformations to prove that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is invariant in two inertial frames.

Sol :- Consider two frames S and S'; the latter moving with velocity v relative to former, such that $v = \hat{i} v_x + \hat{j} v_y + \hat{k} v_z$.

Let the coordinates of two points in frame S be (x_1, y_1, z_1) and (x_2, y_2, z_2); while those in frame S' be (x_1', y_1', z_1') and (x_2', y_2', z_2') From Galilean transformations, we have

$$\left.\begin{array}{l} x_1' = x_1 - v_x t, \quad y_1' = y_1 - v_y t, \quad z_1' = z_1 - v_z t \\ x_2' = x_2 - v_x t, \quad y_2' = y_2 - v_y t, \quad z_2' = z_2 - v_z t \end{array}\right\} \quad (1)$$

The distance between two Points in frame S'

$$= \sqrt{(x_2' - x_1') + (y_2' - x_1') + (z_2' - z_1')}$$

Substituting values of x_1', x_2', y_1', y_2', z_1' and z_2' from (1), we get the distance between two Points in S'

$$= \sqrt{[\{(x_2 - v_x t) - (x_1 - v_x t)\}^2 + \{(y_2 - v_y t) - (y_1 - v_y t)\}^2 + \{(z_2 - v_z t) - (z_1 - v_z t)\}^2]}$$

$$= \sqrt{[\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}]}$$

= The distance between two Points in System S.

Thus we may say That the distance between any two points is invariant under Galilean transformations.